

Gravitating Sphaleron-Antisphaleron Systems

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We present new classical solutions of Einstein-Yang-Mills-Higgs theory, representing gravitating sphaleron-antisphaleron pair, chain and vortex ring solutions. In these static axially symmetric solutions, the Higgs field vanishes on isolated points on the symmetry axis, or on rings centered around the symmetry axis. We compare these solutions to gravitating monopole-antimonopole systems, associating monopole-antimonopole pairs with sphalerons.

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I. INTRODUCTION

The non-trivial topology of the configuration space of the bosonic sector of the $SU(2) \times U(1)$ electroweak theory gives rise to a plethora of unstable classical solutions. Besides the Klinkhamer-Manton sphaleron [1, 2], Weinberg-Salam theory also allows for multisphalerons, which possess either axial or platonic symmetries [3, 4]. Moreover sphaleron-antisphaleron pairs are present [5], and, as shown recently, also sphaleron-antisphaleron chains and vortex rings [6].

When gravity is coupled to this Yang-Mills-Higgs (YMH) theory, the flat space sphaleron changes smoothly, and a branch of gravitating sphalerons arises [7, 8, 9]. This branch bifurcates at a maximal value of the gravitational coupling constant with a second branch, higher in energy. In the limit of vanishing coupling constant, this second branch ends at the lowest Bartnik-McKinnon (BM) solution [10] of Einstein-Yang-Mills (EYM) theory.

Here we consider the effect of gravity on the axially symmetric multisphalerons, and the sphaleron-antisphaleron pairs, chains and vortex rings. We characterize these solutions by two integers, m and n . The Klinkhamer-Manton sphaleron has $m = n = 1$, while the multisphalerons, representing superpositions of n sphalerons, have $m = 1, n > 1$. Sphaleron-antisphaleron pairs are obtained for $m = 2, n = 1, 2$, and chains for $m > 2, n = 1, 2$, while vortex rings arise for $m > 1, n > 2$.

We find, that all these unstable gravitating solutions show the same general coupling constant dependence, as observed for the single gravitating sphaleron. Two branches of solutions arise, a lower branch connected to the flat space solution and an upper branch connected to an EYM solution. For $m = 1$ these EYM solutions correspond to the spherically symmetric BM solution ($n = 1$) or their axially symmetric generalizations ($n > 1$) [10, 11], whereas for $m \geq 2$ ($n \geq 4$) these limiting EYM solutions are of a different type [12]. At the same time additional branches of solutions arise, which connect to the generalized BM solutions [11].

All these solutions thus show a gravity dependence, similar to the monopole-antimonopole pair, chain and vortex ring solutions [13, 14], encountered in the Georgi-Glashow model coupled to gravity, where the Higgs field is not in the fundamental representation of $SU(2)$, but instead in the adjoint representation. We therefore here address the analogy of both sets of solutions. In particular, we find major agreement concerning their general pattern, when we compare sphalerons and sphaleron-antisphaleron systems, characterized by the integers m and n , with monopole-antimonopole systems, characterized by the integers $2m$ and n .

In section II we briefly review the action of $SU(2)$ EYM theory. We present the static axially symmetric Ansätze and the boundary conditions for the solutions in section III. In section IV we then present our numerical results for sphaleron-antisphaleron pairs, chains and vortex rings, and discuss the physical properties of these solutions. We give our conclusions in section V.

II. EINSTEIN-YANG-MILLS-HIGGS THEORY

We consider $SU(2)$ Einstein-Yang-Mills-Higgs theory with action

$$S = \int \left\{ \frac{R}{16\pi G} - \frac{1}{2} \text{Tr} (F_{\mu\nu} F^{\mu\nu}) - (D_\mu \Phi)^\dagger (D^\mu \Phi) - \lambda (\Phi^\dagger \Phi - \frac{v^2}{2})^2 \right\} \sqrt{-g} d^4x, \quad (1)$$

with curvature scalar R , $\text{su}(2)$ field strength tensor

$$F_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + ie[V_\mu, V_\nu] , \quad (2)$$

$\text{su}(2)$ gauge potential $V_\mu = V_\mu^a \tau_a / 2$, and covariant derivative of the Higgs Φ in the fundamental representation

$$D_\mu \Phi = (\partial_\mu + ieV_\mu) \Phi , \quad (3)$$

where G and e denote the gravitational and gauge coupling constants, respectively, λ denotes the strength of the Higgs self-interaction and v the vacuum expectation value of the Higgs field.

The action (1) is invariant under local $SU(2)$ gauge transformations U ,

$$\begin{aligned} V_\mu &\longrightarrow UV_\mu U^\dagger + \frac{i}{e} \partial_\mu UU^\dagger , \\ \Phi &\longrightarrow U\Phi . \end{aligned}$$

The gauge symmetry is spontaneously broken due to the non-vanishing vacuum expectation value of the Higgs field

$$\langle \Phi \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} , \quad (4)$$

leading to the vector and scalar boson masses

$$M_W = \frac{1}{2}ev , \quad M_H = v\sqrt{2\lambda} . \quad (5)$$

In the absence of gravity, the Lagrangian reduces to the bosonic sector of Weinberg-Salam theory with vanishing Weinberg angle. Setting the Weinberg angle to zero is a good approximation for sphalerons and multisphalerons [3]. We here consider gravitating sphaleron, sphaleron-antisphaleron chain and vortex ring solutions in the limit of vanishing Weinberg angle.

In the standard model fermion number is not conserved [15]. Reexpressing the anomaly term in terms of the Chern-Simons current

$$K^\mu = \frac{e^2}{16\pi^2} \varepsilon^{\mu\nu\rho\sigma} \text{Tr}(F_{\nu\rho} V_\sigma + \frac{2}{3} ie V_\nu V_\rho V_\sigma) , \quad (6)$$

yields for the fermion charge of a sphaleron solution (in a suitable gauge) [1]

$$Q_F = \int d^3r K^0 . \quad (7)$$

III. ANSATZ AND BOUNDARY CONDITIONS

To obtain gravitating static axially symmetric solutions, we employ isotropic coordinates [11]. In terms of the spherical coordinates r , θ and φ the isotropic metric reads

$$ds^2 = -f dt^2 + \frac{h}{f} dr^2 + \frac{hr^2}{f} d\theta^2 + \frac{lr^2 \sin^2 \theta}{f} d\varphi^2 , \quad (8)$$

where the metric functions f , h and l are functions of the coordinates r and θ , only. The z -axis ($\theta = 0, \pi$) represents the symmetry axis. Regularity on this axis requires

$$h|_{\theta=0,\pi} = l|_{\theta=0,\pi} . \quad (9)$$

We take a purely magnetic gauge field, $V_0 = 0$, and parametrize the gauge potential and the Higgs field by the Ansatz [6, 16]

$$V_i dx^i = \left(\frac{H_1}{r} dr + (1 - H_2) d\theta \right) \frac{\tau_\varphi^{(n)}}{2e} - n \sin \theta \left(H_3 \frac{\tau_r^{(n,m)}}{2e} + (1 - H_4) \frac{\tau_\theta^{(n,m)}}{2e} \right) d\varphi , \quad V_0 = 0 , \quad (10)$$

and

$$\Phi = i(\Phi_1 \tau_r^{(n,m)} + \Phi_2 \tau_\theta^{(n,m)}) \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (11)$$

where

$$\begin{aligned} \tau_r^{(n,m)} &= \sin m\theta (\cos n\varphi \tau_x + \sin n\varphi \tau_y) + \cos m\theta \tau_z, \\ \tau_\theta^{(n,m)} &= \cos m\theta (\cos n\varphi \tau_x + \sin n\varphi \tau_y) - \sin m\theta \tau_z, \\ \tau_\varphi^{(n)} &= (-\sin n\varphi \tau_x + \cos n\varphi \tau_y), \end{aligned}$$

n and m are integers, and τ_x , τ_y and τ_z denote the Pauli matrices.

The two integers n and m determine the fermion number of the solutions [4, 6],

$$Q_F = \frac{n(1 - (-1)^m)}{4}. \quad (12)$$

For vanishing gravity and $m = n = 1$ the Ansatz yields the Klinkhamer-Manton sphaleron [1]. For $n > 1$ or $m > 1$, the functions H_1, \dots, H_4 , Φ_1 , and Φ_2 depend on r and θ , only. These axially symmetric solutions represent gravitating multisphaleron ($m = 1, n > 2$), sphaleron-antisphaleron pair ($m = 2, n = 1$), chain ($m > 2, n = 1$), and vortex ring ($m > 1, n > 2$) configurations as well as mixed configurations.

With this Ansatz the full set of field equations reduces to a system of nine coupled partial differential equations in the independent variables r and θ . A residual $U(1)$ gauge degree of freedom is fixed by the condition $r\partial_r H_1 - \partial_\theta H_2 = 0$ [2].

Regularity and finite energy require the boundary conditions

$$\begin{aligned} r = 0 : \quad & \partial_r f(r, \theta) = \partial_r h(r, \theta) = \partial_r l(r, \theta) = 0, \\ & H_1 = H_3 = 0, \quad H_2 = H_4 = 1, \\ & \Phi_1 = \Phi_2 = 0, \\ r \rightarrow \infty : \quad & f = h = l = 1, \\ & H_1 = H_3 = 0, \quad H_2 = 1 - 2m, \quad 1 - H_4 = \frac{2 \sin m\theta}{\sin \theta}, \\ & \Phi_1 = 1, \quad \Phi_2 = 0, \\ \theta = 0, \pi : \quad & \partial_\theta f = \partial_\theta h = \partial_\theta l = 0 \\ & H_1 = H_3 = 0, \quad \partial_\theta H_2 = \partial_\theta H_4 = 0, \\ & \partial_\theta \Phi_1 = 0, \quad \Phi_2 = 0, \end{aligned} \quad (13)$$

for odd m , while at $r = 0$ ($\sin m\theta \Phi_1 + \cos m\theta \Phi_2$) = 0, $\partial_r(\cos m\theta \Phi_1 - \sin m\theta \Phi_2) = 0$ is required for even m .

We note that the gauge potential approaches a pure gauge at infinity,

$$A_\mu \rightarrow \frac{i}{e}(\partial_\mu U)U^\dagger, \quad U = e^{-ik\theta\tau_\varphi^{(n)}}, \quad (14)$$

when the Higgs field is in the doublet representation ($m = k$), and likewise for monopole-antimonopole systems with vanishing total magnetic charge ($m = 2k$), and for pure EYM solutions.

Let us now introduce the dimensionless coordinate x and the dimensionless coupling constant α

$$x = \frac{e\alpha}{\sqrt{4\pi G}} r, \quad \alpha = \sqrt{4\pi G} v. \quad (15)$$

The limit $\alpha \rightarrow 0$ can be approached in two different ways: 1. $G \rightarrow 0$, while the Higgs vacuum expectation value v remains finite (flat-space limit), and 2. $v \rightarrow 0$, while Newton's constant G remains finite. These limits are then associated with different branches of solutions.

The dimensionless mass M of the solutions is obtained from the asymptotic expansion of the metric function f ,

$$M = \frac{1}{2\alpha^2} \lim_{x \rightarrow \infty} x^2 \partial_x f = \frac{\mu}{\alpha^2}. \quad (16)$$

IV. RESULTS

Let us first briefly recall the new static axially symmetric solutions of Weinberg-Salam theory (in the limit of vanishing Weinberg angle), found recently [6]. We here restrict the discussion to the case of vanishing Higgs mass. The axially symmetric solutions are characterized by two integers, m and n . (We do not consider radial excitations of the solutions, associated with a third integer.) The electroweak sphaleron corresponds to the special case $m = n = 1$, multisphalerons are obtained for $m = 1, n > 1$, while the new electroweak solutions require $m > 1$. Like the electroweak sphaleron the new solutions are unstable, corresponding to saddle points.

For $n \leq 2$, the modulus of the Higgs field of these solutions vanishes on m discrete points on the symmetry axis, thus these flat space solutions correspond to sphaleron-antisphaleron pairs and chains. For $n > 2$ the solutions change character, and the modulus of the Higgs field vanishes on one or more rings centered around the symmetry axis. While $n = 3$ and 4 represent a transitional regime, where (two or more) isolated nodes on the symmetry axis and rings coexist, for larger values of n these solutions possess $[m/2]$ vortex rings (where $[m/2]$ denotes the integer part of $m/2$). For even m , these solutions are thus pure electroweak vortex ring solutions, while for odd m these solutions represent electroweak sphaleron-vortex ring superpositions, since here an isolated node at the origin is retained.

Let us now consider the coupling to gravity, while restricting to vanishing Higgs coupling constant, $\lambda = 0$. In Fig. 1 we exhibit the scaled mass μ/α of the single sphaleron ($m = n = 1$) and the multisphalerons ($m = 1, n = 2, \dots, 5$) versus the coupling constant α . Whereas the mass M is finite in the flat space limit $\alpha = 0$, the scaled mass $\alpha M = \mu/\alpha$ vanishes there.

In each case, a lower branch of gravitating sphalerons resp. multisphalerons emerges from the corresponding flat space solution at $\alpha = 0$, and extends up to a maximal value of the coupling constant, α_{\max} , beyond which no such globally regular solutions exist. At α_{\max} the lower branch merges with a second branch, the upper branch, which extends back to $\alpha \rightarrow 0$. The mass M diverges on the upper branch in the limit $\alpha \rightarrow 0$. The scaled mass μ/α , in contrast, assumes a finite limiting value. Rescaling the solutions then shows, that in the limit $\alpha \rightarrow 0$ in each case a globally regular EYM solution is reached, corresponding to the first BM solution ($n = 1$) or a generalized BM solution ($n > 1$) [10, 11]. (Note, that the $n = 1$ gravitating sphaleron was studied before [7, 8, 9].)

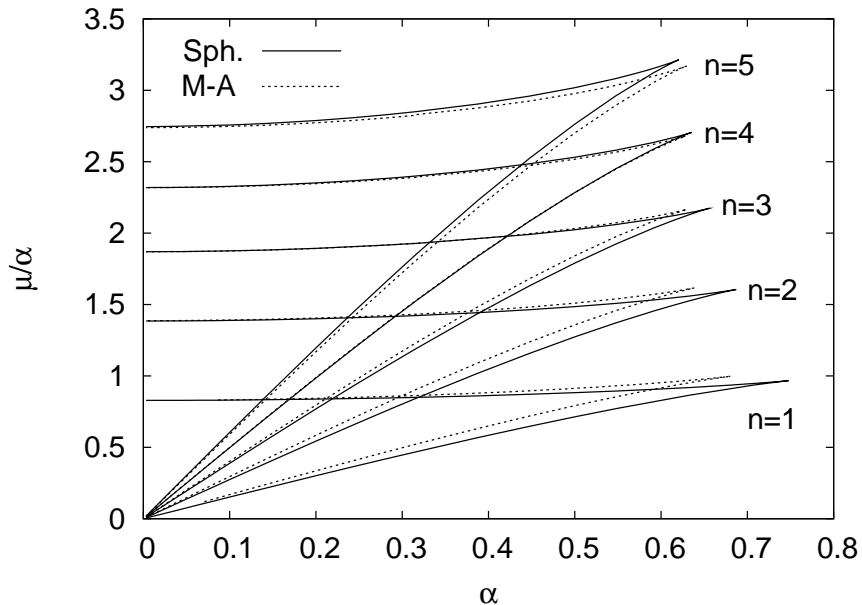


Figure 1: Scaled mass μ/α versus coupling constant α for the single sphaleron ($m = 1, n = 1$) and multisphaleron ($m = 1, n = 2, \dots, 5$) solutions; for comparison the mass of the monopole-antimonopole pair ($m = 2, n = 1, 2$) and vortex ring ($m = 2, n = 3, 5$) solutions is also shown.

Considering the sets of solutions for different values of n , we note, that with increasing n the maximal value of the coupling constant α_{\max} decreases monotonically. At the same time the maximal value of the scaled mass increases monotonically and almost linearly.

We now compare these gravitating sphaleron and multisphaleron solutions with gravitating monopole-antimonopole pair and vortex ring solutions [13, 14]. In these solutions the Higgs field is in the triplet representation, thus providing only two of the gauge bosons with mass, while the third remains massless and is therefore associated with the

electromagnetic field. The monopole-antimonopole pair and vortex ring solutions with $m = 2$ are magnetically neutral, but possess magnetic dipole moments, just like the sphaleron solutions at finite Weinberg angle [1, 2, 17].

The figure shows, that the monopole-antimonopole pair and vortex ring solutions with $m = 2$ exhibit precisely the same pattern as the sphaleron and multisphalerons solutions with $m = 1$, when their dependence on the coupling constant α and the integer n is considered. Interestingly, for the case $n = 4$, the agreement of the scaled mass is excellent on both branches of solutions, which therefore almost coincide. Whether this is accidental, or whether there is a hidden reason for this remarkable agreement is not clear at the moment, though.

Let us next consider larger values of m , i.e., consider the dependence of sphaleron-antisphaleron pairs and vortex rings on the coupling constant α . In Fig. 2 we exhibit the scaled mass μ/α of the gravitating sphaleron ($m = n = 1$) together with the scaled mass of the gravitating sphaleron-antisphaleron pair ($m = 2, n = 1$) and chain ($m = 3, n = 1$) versus the coupling constant α . Again, the mass M is finite in the flat space limit $\alpha = 0$, while the scaled mass $\alpha M = \mu/\alpha$ vanishes there.

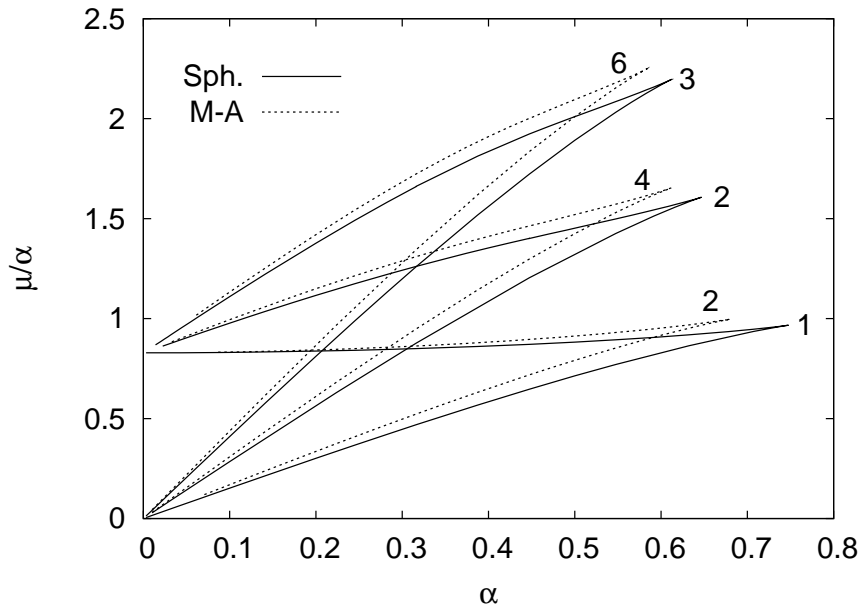


Figure 2: Scaled mass μ/α versus coupling constant α for single sphaleron ($m = 1, n = 1$), sphaleron-antisphaleron pair ($m = 2, n = 1$), and chain ($m = 3, n = 1$) solutions; for comparison the mass of the monopole-antimonopole pair ($m = 2, n = 1$), and chain ($m = 4, n = 1$), ($m = 6, n = 1$) solutions is also shown.

Also for the sphaleron-antisphaleron pair ($m = 2$) and chain ($m = 3$) a lower branch of gravitating solutions emerges from the corresponding flat space solution at $\alpha = 0$, and extends up to a maximal value of the coupling constant, α_{\max} , where it merges with a second branch, the upper branch, which extends back to $\alpha \rightarrow 0$. In the limit $\alpha \rightarrow 0$, the scaled mass μ/α assumes the same finite limiting value for the sphaleron-antisphaleron pair and chain as for the single sphaleron. Clearly, in all cases (after rescaling) the first BM solution ($n = 1$) is reached [10].

Associating again these gravitating sphaleron-antisphaleron solutions with the corresponding gravitating monopole-antimonopole systems, we must resort to $m = 4$ and $m = 6$ monopole-antimonopole chains, respectively. This is clear, because we already associated a monopole-antimonopole pair with a single sphaleron. Thus we must associate two monopole-antimonopole pairs with the sphaleron-antisphaleron pair, and three with the sphaleron-antisphaleron ($m = 3$) chain. These then represent monopole-antimonopole chains composed of $m = 4$ and $m = 6$ constituents, respectively, i.e. m monopoles and antimonopoles, alternating on the symmetry axis.

The figure again reveals, that the monopole-antimonopole chains with $m = 4$ and $m = 6$ exhibit precisely the same pattern as the sphaleron-antisphaleron pairs ($m = 2$) and chains ($m = 3$), respectively, when their dependence on the coupling constant α is considered. In general, we conclude, that sphaleron-antisphaleron chains with m constituents, can be associated with monopole-antimonopole chains, composed of $2m$ constituents.

Let us finally consider gravitating sphaleron-antisphaleron vortex rings for $m = 2$ and $n = 4$. In Fig. 3 we exhibit for these solutions the scaled mass μ/α versus the coupling constant α , together with the scaled mass of the gravitating multisphalerons ($m = 1, n = 4$). For these gravitating sphaleron-antisphaleron vortex rings we observe four instead of two branches of solutions. As before the lower flat space branch ends and merges with the upper branch at α_{\max} . The upper branch, however, does not connect to the generalized BM solution in the limit $\alpha \rightarrow 0$ [11]. Instead it reaches

an EYM solution of a different type, which exists only when $m \geq 2$ and $n \geq 4$ [12].

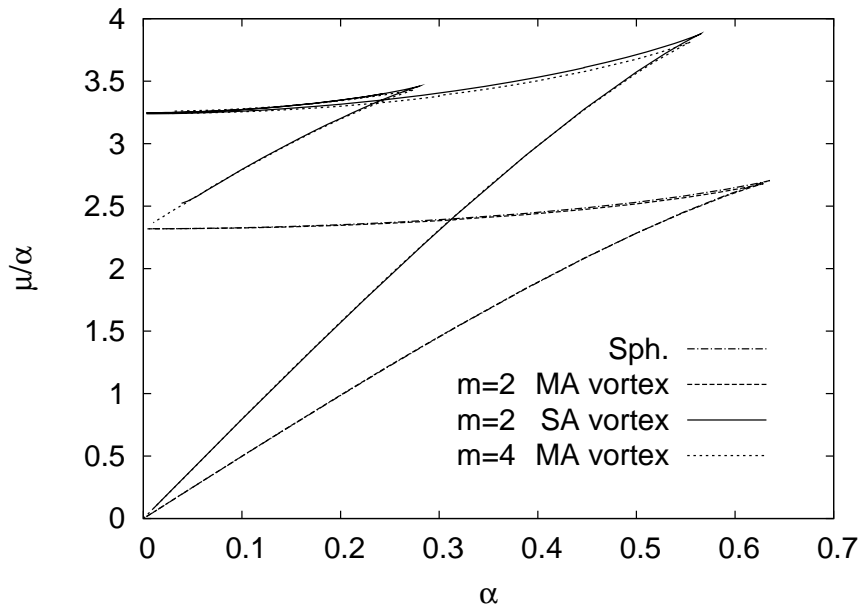


Figure 3: Scaled mass μ/α versus coupling constant α for single sphaleron ($m = 1$, $n = 4$) and sphaleron-antisphaleron vortex ring ($m = 2$, $n = 4$) solutions; for comparison the mass of monopole-antimonopole ($m = 2$, $n = 4$) and monopole-antimonopole ($m = 4$, $n = 4$) vortex ring solutions is also shown.

Since EYM solutions of this type always come in pairs for a given set of integers m and n , a second $n = 4$ EYM solution is present, which is slightly higher in mass [12]. This second EYM solution constitutes the endpoint of a second upper branch of sphaleron-antisphaleron solutions, which merges at a second critical value of the coupling constant α_{cr} with a second lower branch, and it is along this second lower branch, that in the limit $\alpha \rightarrow 0$ the solutions connect to the generalized BM solution with $n = 4$.

We now compare these gravitating $m = 2$ sphaleron-antisphaleron vortex ring solutions again with the corresponding gravitating $m = 4$ monopole-antimonopole vortex ring solutions. As anticipated, the figure reveals, that the monopole-antimonopole vortex rings with $m = 4$ exhibit precisely the same pattern as the sphaleron-antisphaleron vortex rings with $m = 2$, when their dependence on the coupling constant α is considered. Moreover, the masses show again an intriguing quantitative agreement for these $n = 4$ solutions, which remains to be understood.

The sphaleron-antisphaleron chain and vortex ring configurations presented above were all obtained for vanishing Higgs mass. As a function of the Higgs mass these solutions also form branches [6]. At critical values of the Higgs mass bifurcations arise, where new branches of solutions appear. These then give rise to a plethora of further gravitating solutions to be discussed elsewhere.

V. CONCLUSIONS

We have investigated gravitating sphalerons, multisphalerons and sphaleron-antisphaleron systems, which are static and axially symmetric, and characterized by two integers, m and n . Single sphalerons are obtained for $m = n = 1$, multisphalerons for $m = 1$ and $n > 1$, and sphaleron-antisphaleron systems for $m > 1$. Like the electroweak sphaleron these new solutions are unstable, corresponding to saddle points.

In the presence of gravity, from each of these flat space solutions, a branch of gravitating solutions emerges and evolves smoothly with increasing gravitational coupling constant α up to a maximal value α_{max} . There it merges with a second branch, higher in energy, which extends backwards to $\alpha = 0$. In the limit, the Higgs vacuum expectation value vanishes, and the limiting solutions correspond to pure EYM solutions (after rescaling).

For larger values of the Higgs mass, the flat space solutions are no longer uniquely specified by the integers m and n . Instead bifurcations appear, giving rise to further branches and types of configurations. As for the monopole-antimonopole systems [14], we therefore expect a plethora of gravitating solutions at large scalar coupling. Furthermore, for very large values of the Higgs mass also bisphalerons or ‘deformed’ sphalerons are present [9, 18], which do not exhibit parity reflection symmetry.

Comparing these gravitating sphaleron, multisphaleron and sphaleron-antisphaleron solutions, based on a doublet Higgs field, with the monopole-antimonopole solutions, obtained with a triplet Higgs field, we find precisely the same pattern of branches of solutions, when we compare sphalerons and sphaleron-antisphaleron systems characterized by m and n , with monopole-antimonopole systems characterized by $2m$ and n . Interestingly, in the case $n = 4$, the scaled mass of both types of solutions even almost coincides.

Monopole-antimonopole systems can rotate, when they carry no magnetic charge [19]. It therefore appears interesting to consider also rotating sphaleron-antisphaleron systems. Moreover, monopole-antimonopole systems can be endowed with a black hole at their center, as shown explicitly already for the monopole-antimonopole pair [20]. Sphaleron-antisphaleron systems with black holes are thus expected to exist as well.

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